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TYPE CONSTANTS AND $(q,2)$-SUMMING NORMS DEFINED BY n VECTORS

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ABSTRACT

For $q \geqq 2$, the $(q,2)$-summing norm of an operator of rank n can be computed, up to a constant c_q, by an appropriate choice of at most n vectors. A corresponding statement is true for the Gaussian type and cotype constants of n-dimensional spaces.

A linear operator $T: X \to Y$ between Banach spaces is said to be $(q,2)$-absolutely summing, where $q \geqq 2$, if there is $M > 0$ such that for any finite sequence $(x_i)_{i=1}^n \subseteq X$

$$\left(\sum_{i=1}^n \| Tx_i \|^q\right)^{1/q} \leqq M \sup_{\|x'\|_{X^*} \leqq 1} \left(\sum_{i=1}^n |\langle x', x_i\rangle|^2\right)^{1/2}.$$

The infimum over all possible values of M is the $(q,2)$-summing norm, denoted by $\pi_{q,2}(T)$. If only sequences $(x_i)_{i=1}^n$ of fixed length n are considered, the infimum over all M is denoted by $\pi_{q,2}^{(n)}(T)$. $\Pi_{q,2}(X,Y)$ stands for all $(q,2)$-summing operators from X into Y.

Let g_i be a sequence of independent normalized Gaussian random variables on a probability space (Ω, P). Given a Banach space X, we let, for any $n \in \mathbf{N}$ and $1 \leqq p \leqq 2 \leqq q \leqq \infty$, $K^{(p,n)}(X)$ and $K_{(q,n)}(X)$ be the smallest constants for which

$$K_{(q,n)}(X)^{-1}\left(\sum_{i=1}^n \|x_i\|^q\right)^{1/q} \leqq \left(\int_\Omega \left\|\sum_{i=1}^n g_i(\omega)x_i\right\|^2 dP(\omega)\right)^{1/2} \leqq K^{(p,n)}(X)\left(\sum_{i=1}^n \|x_i\|^p\right)^{1/p} \tag{1}$$

for every choice of $(x_i)_{i=1}^n \subseteq X$. If the left (respectively the right) inequality in (1) holds for any $n \in \mathbf{N}$, X is of Gaussian cotype q (respectively Gaussian type p); the corresponding constants are denoted by $K_{(q)}(X)$ ($K^{(p)}(X)$).

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Clearly $\pi_{q,2}^{(n)}(T) \leqq \pi_{q,2}(T)$, $K_{(q,n)}(X) \leqq K_{(q)}(X)$ and $K^{(p,n)}(X) \leqq K^{(p)}(X)$ for any $p \leqq 2 \leqq q$, $n \in \mathbf{N}$, T and X. On the other hand, Tomczak-Jaergermann [6] showed that the (2, 2)-summing norm of any rank n operator T essentially can be calculated by n vectors, i.e.

$$\pi_{2,2}(T) \leqq 2\pi_{2,2}^{(n)}(T)$$

and similarly for the (co)type 2 constants of n-dimensional spaces X,

$$K^{(2)}(X) \leqq 2K^{(2,n)}(X), \qquad K_{(2)}(X) \leqq 2K_{(2,n)}(X).$$

The aim of this note is to derive corresponding inequalities for $p \leqq 2$ and $q \geqq 2$. We prove the

THEOREM. *For any $q > 2$, there is a constant c_q such that for any operator $T: X \to Y$ of rank n*

$$\pi_{q,2}(T) \leqq c_q \pi_{q,2}^{(n)}(T).$$

There is an absolute constant c such that $c_q \leqq c/(q-2)$.

Whereas the proof of [6] does not work for $q > 2$, the proof of the theorem given here does not apply in the case $q = 2$. It is unknown whether c_q can be chosen to be bounded as q tends to 2. By the method of [6], theorem 1 implies

COROLLARY 1. *Let $1 < p \leqq 2 \leqq q < \infty$, $p' = p/(p-1)$. Let X be a n-dimensional space. Then*

$$K^{(p,n)}(X) \leqq K^{(p)}(X) \leqq c_{p'} K^{(p,n)}(X),$$

$$K_{(q,n)}(X) \leqq K_{(q)}(X) \leqq c_q K_{(q,n)}(X).$$

The proof of the theorem uses three lemmas. We need some notation first. The approximation numbers of an operator $T: X \to Y$ are given by

$$\alpha_j(T) := \inf\{\|T - T_j\| \mid T_j : X \to Y \text{ of rank} < j\}, \quad j = 1, 2, \cdots.$$

For any $0 < q \leqq \infty$, we let

$$S_q(X, Y) := \left\{ T : X \to Y \mid \sigma_q(T) := \left(\sum_{j=1}^{\infty} \alpha_j(T)^q \right)^{1/q} < \infty \right\},$$

with $S_\infty(X, Y) :=$ all continuous linear maps, and

$$S_{2,1}(X, Y) := \left\{ T : X \to Y \mid \sigma_{2,1}(T) := \sum_{j=1}^{\infty} \alpha_j(T) j^{-1/2} < \infty \right\};$$

σ_q is a quasinorm on $S_q(X, Y)$. $S_{2,1}(X, Y)$ corresponds to the Lorentz sequence space

$$l_{2,1} := \left\{ (\xi_j)_{j=1}^{\infty} \mid \xi_j \in \mathbf{K}, \ \|\xi\|_{2,1} := \sum_{j=1}^{\infty} \xi_j^* j^{-1/2} < \infty \right\}.$$

Here ξ_j^* is the (decreasing) rearrangement of ξ_j.

LEMMA 1. *For any* $T : l_2^n \to X$ *and* $q \geqq 2$, $\sigma_q(T) \leqq \pi_{q,2}^{(n)}(T)$.

PROOF. This is a modified form of a lemma of Lewis [4]. We define inductively an orthonormal basis $(e_j)_{j=1}^n$ of l_2^n with

$$\alpha_j(T) \leqq \|Te_j\|. \tag{2}$$

For $j = 1$, choose $e_1 \in l_2^n$ of norm 1 such that $\alpha_j(T) = \|T\| = \|Te_1\|$. If $j < n$ orthonormal vectors $e_1, \cdots, e_j$ with (2) have been constructed, let $Y_j := [e_1, \cdots, e_j]$ and $P_j : l_2^n \to Y_j \subseteq l_2^n$ be the orthonormal projection. Since $\operatorname{rank} P_j = j$,

$$\alpha_{j+1}(T) \leqq \|T - TP_j\| = \|T|_{Y_j^{\perp}}\|.$$

Hence there is $e_{j+1} \in Y_j^{\perp}$ with $\|e_{j+1}\| = 1$ such that $\alpha_{j+1}(T) \leqq \|Te_{j+1}\|$, the vector for $j+1$. Since $\operatorname{rank} T \leqq n$, one has $\alpha_k(T) = 0$ for $k > n$. Hence by (2)

$$\sigma_q(T) = \left(\sum_{j=1}^{n} \alpha_j(T)^q \right)^{1/q} \leqq \left(\sum_{j=1}^{n} \|Te_j\|^q \right)^{1/q} \leqq \pi_{q,2}^{(n)}(T). \qquad \blacksquare$$

LEMMA 2. *For any* X *and* Y, $S_{2,1}(X, Y) \subseteq \Pi_{2,2}(X, Y)$.

PROOF. Take any operator $T \in S_{2,1}(X, Y)$. Choose $D_j : X \to Y$ of rank $D_j < 2^j$ such that $\|T - D_j\| \leqq 2\alpha_{2^j}(T)$, $j = 0, 1, \cdots$ $(D_0 = 0)$. Let $T_j = D_{j+1} - D_j$. Then $T = \sum_{j=0}^{\infty} T_j$, $\|T_j\| \leqq 4\alpha_{2^j}(T)$ and $k_j := \operatorname{rank} T_j < 2^{j+2}$. By Garling–Gordon [1], the 2-summing norm of the identity map I_j on the k_j-dimensional space T_jX is $\pi_{2,2}(I_j) = \sqrt{k_j}$. Hence

$$\pi_{2,2}\left(\sum_{j=0}^{N} T_j \right) \leqq \sum_{j=0}^{N} \pi_{2,2}(T_j) \leqq \sum_{j=0}^{N} \|T_j\| \, \pi_{2,2}(I_j)$$

$$\leqq 2 \sum_{j=0}^{N} 2^{j/2} \|T_j\| \leqq 16 \sum_{j=0}^{N} 2^{j/2-1} \alpha_{2^j}(T)$$

which, using the monotonicity of the approximation numbers, is

$$\leqq 16 \sum_{k=1}^{\infty} k^{-1/2} \alpha_k(T) = 16 \sigma_{2,1}(T),$$

which is bounded independent of N. Thus T is 2-summing and $\pi_{2,2}(T) \leqq 16\sigma_{2,1}(T)$. ■

Let Z_1 and Z_2 be (quasi)normed spaces with $Z_1 \subseteq Z_2$. Recall that the K-functional for (Z_1, Z_2) is given by

$$K(t, z; Z_1, Z_2) := \inf\{\|z_1\|_{Z_1} + t\|z_2\|_{Z_2} \mid z = z_1 + z_2, z_1 \in Z_1, z_2 \in Z_2\}$$

for $t \in \mathbf{R}^+$ and $z \in Z_2$. For $0 < q \leqq \infty$, $0 < \theta < 1$, the real interpolation space

$$(Z_1, Z_2)_{\theta,q} := \left\{ z \in Z_2 \,\Big|\, \|z\|_{\theta,q} = \left(\int_0^\infty (K(t, z; Z_1, Z_2) t^{-\theta})^q \frac{dt}{t} \right)^{1/q} < \infty \right\}$$

is again a quasinormed space, with quasinorm $\|\cdot\|_{\theta,q}$.

LEMMA 3. (a) *There are absolute constants* c_1, c_2 *such that for any* $T: X \to Y$ *and* $t \in \mathbf{R}^+$

$$c_1 K(t, T; S_{2,1}(X, Y), S_\infty(X, Y)) \leqq K(t, (\alpha_j(T))_{j=1}^\infty; l_{2,1}, l_\infty)$$
$$\leqq c_2 K(t, T; S_{2,1}(X, Y), S_\infty(X, Y)).$$

(b) *For any* X, Y *and* $2 < q < \infty$ *with* $1/q = (1-\theta)/2$

$$S_q(X, Y) = (S_{2,1}(X, Y), S_\infty(X, Y))_{\theta,q},$$

$$\Pi_{q,2}(X, Y) \subseteq (\Pi_{2,2}(X, Y), \Pi_{\infty,2}(X, Y))_{\theta,q},$$

and with absolute constants c_1, c_2, c_3

$$c_1 \sigma_q(T) \leqq \|T\|_{(S_{2,1}, S_\infty)_{\theta,q}} \leqq c_2/(q-2)\sigma_q(T), \tag{3}$$

$$\|T\|_{(\Pi_{2,2}, \Pi_{\infty,2})_{\theta,q}} \leqq c_3 \pi_{q,2}(T). \tag{4}$$

Here $\pi_{\infty,2}(X, Y) = S_\infty(X, Y)$ *denotes all continuous linear maps.*

PROOF. Let $0 < p_1 < p_2 \leqq \infty$ and X and Y be Banach spaces. By proposition 1 of [3]

$$K(t, T; S_{p_1}(X, Y), S_{p_2}(X, Y)) \sim K(t, (\alpha_j(T))_{j=1}^\infty; l_{p_1}, l_{p_2}). \tag{5}$$

Here $\sim$ indicates equivalence up to constants depending only on p_1 and p_2 which are bounded as $p_2 \to \infty$. The proof of (5) given in [3] by mistake only works in the case $p_2 = \infty$ (which we need). However, the case $p_2 < \infty$ follows easily from theorem 2.1 and remark 2.1 of T. Holmstedt [2] and the equivalence for $p_2 = \infty$. It is well-known [2] that $(l_1, l_\infty)_{1/2,1} = l_{2,1}$. Thus using (5) for $p_1 = 1$, $p_2 = \infty$ we get

$$(S_1(X, Y), S_\infty(X, Y))_{1/2,1} = S_{2,1}(X, Y).$$

A twofold application of theorem 2.1 of [2] now yields

$$K(t,T;S_{2,1},S_\infty)\sim\int_0^{t^2}s^{-1/2}K(s,T;S_1,S_\infty)ds/s$$

$$\sim\int_0^{t^2}s^{-1/2}K(s,(\alpha_j(T))_{j=1}^\infty;l_1,l_\infty)ds/s$$

$$\sim K(t,(\alpha_j(T))_{j=1}^\infty;l_{2,1},l_\infty).$$

Hence by definition of the interpolation spaces

$$\|T\|_{(S_{2,1},S_\infty)_{\theta,q}}\sim\|(\alpha_j(T))_{j=1}^\infty\|_{(l_{2,1},l_\infty)_{\theta,q}}\sim\|(\alpha_j(T))_{j=1}^\infty\|_{l_q}=\sigma_q(T)$$

for $q>2$, $1/q=(1-\theta)/2$, using $l_q=(l_{2,1},l_\infty)_{\theta,q}$. The dependence of the constants in (3) on q follows from these estimates and theorem 3.1 of [2].

Finally (4) follows easily from $l_q=(l_2,l_\infty)_{\theta,q}$, cf. [3]. ■

PROOF OF THE THEOREM. Let $S:X\to Y$ be of rank n. It is a consequence of

$$\pi_{q,2}(S)=\sup\{\pi_{q,2}(SA)\mid A:l_2\to X \text{ of } \|A\|\leqq 1\},$$

that it suffices to prove Theorem 1 for maps $T:l_2^n\to X$. Let $q>2$ and $1/q=(1-\theta)/2$. By Lemmas 2 and 3,

$$\begin{aligned}S_q(l_2,X)&=(S_{2,1}(l_2,X),S_\infty(l_2,X))_{\theta,q}\\&\subseteq(\Pi_{2,2}(l_2,X),\Pi_{\infty,2}(l_2,X))_{\theta,q}\qquad(6)\\&\subseteq\Pi_{q,2}(l_2,X)\end{aligned}$$

with (quasi)norm inequality $\pi_{q,2}(R)\leqq c_q\sigma_q(R)$ for any map $R:l_2\to X$ and $c_q\leqq c/(q-2)$. Thus by Lemma 1

$$\pi_{q,2}(T)\leqq c_q\sigma_q(T)\leqq c_q\pi_{q,2}^{(n)}(T).$$ ■

Note that Lemma 1 and (6) actually yield:

COROLLARY 2. *For any $2<q<\infty$ and any Banach space X*

$$\Pi_{q,2}(l_2,X)=S_q(l_2,X).$$

Clearly, this is false for $q=2$ and the reason why the proof of the theorem given does not work for $q=2$.

PROOF OF COROLLARY 1. The following characterization of the Gaussian cotype and type constants for $1<p\leqq 2\leqq q<\infty$ was noted by Tomczak-Jaegermann [6], [7]:

$$K_{(q,n)}(X) = \sup\{\pi_{q,2}^{(n)}(T) \mid T : l_2^n \to X \text{ with } l(T) \leqq 1\},$$

$$K^{(p,n)}(X) = \sup\{l(S) \mid S : l_2^n \to X \text{ with } (\pi_{p',2}^{(n)})^*(S^*) \leqq 1\}.$$

Here $(\pi_{p',2}^{(n)})^*$ denotes the adjoint ideal norm and l the ideal norm

$$l(T) = \left(\int_\Omega \left\| \sum_{i=1}^n g_i(t) Te_i \right\|^2 dP(t) \right)^{1/2}.$$

Similar statements hold for $n = \infty$. These facts and Theorem 1 as well as its adjoint form imply Corollary 1. ∎

By the Pietsch factorization theorem, any map $T : l_2^n \to X$ can be factored as $T = SR$, $S : l_2^n \to X$, $R : l_2^n \to l_2^n$ such that $\sigma_2(R)\|S\| = \pi_2(R)\|S\| = \pi_2(T)$. This factorization is one of the main steps in the proof of $\pi_2(T) \leqq 2\pi_2^{(n)}(T)$ in [6]. It is the reason why the proof of [6] does not generalize to $\pi_{q,2}$, since a corresponding factorization (in)equality $\sigma_q(R)\|S\| = \pi_{q,2}(R)\|S\| \leqq c\pi_{q,2}(T)$ is false in general, although only by a logarithmic factor. The best possible result concerning this problem is the

PROPOSITION. (a) *For any $2 < q < \infty$, there is $c_q > 0$ such that for any n and any rank n operator $T : l_2^n \to X$ there is a factorization $T = SR$, $S : l_2^n \to X$, $R : l_2^n \to l_2^n$ with*

$$\sigma_q(R)\|S\| \leqq c_q(\ln(n+1))^{1/2}\pi_{q,2}(T).$$

(b) *There are operators $T_n : l_2^n \to l_1$ such that for any factorization $T_n = S_nR_n$, $S_n : l_2^n \to l_1$, $R_n : l_2^n \to l_2^n$ one has*

$$\sigma_q(R_n)\|S_n\| \geqq c_q(\ln(n+1))^{1/2}\pi_{q,2}(T_n).$$

Here c_q depends only on q, $2 < q < \infty$.

PROOF. (a) Let N be such that $2^{N-1} \leqq n < 2^N$ and choose again maps $D_j : l_2^n \to X$ of rank $D_j < 2^j$ such that $D_0 = 0$, $D_N = T$ and $\|T - D_j\| \leqq 2\alpha_{2^j}(T)$. Let $T_j = D_j - D_{j-1}$ for $j = 1, \cdots, N$. Then $T = \sum_{j=1}^N T_j$ with $t_j :=$ rank $T_j < 2^{j+1}$ and, as in Lemma 2, one gets

$$\left(\sum_{j=1}^N 2^j \|T_j\|^q \right)^{1/q} \leqq c_q\sigma_q(T) \leqq c_q\pi_{q,2}(T) \tag{7}$$

using Lemma 1. Let P_j denote the orthogonal projection onto the (t_j-dimensional) space $l_2^n \ominus \operatorname{Ker} T_j = l_2^{t_j}$. Taking the l_2-sum of these spaces, we define

$$R : l_2^n \to l_2^N(l_2^{t_j}) =: (l_2^{t_1} \oplus \cdots \oplus l_2^{t_N})_2$$

by $Rx = (\|T_j\| P_j x)_{j=1}^N$. Let $S : l_2^N(l_2^{\prime}) \to X$ be given by $S(\xi_j)_{j=1}^N = \sum_{j=1}^N T_j(\xi_j / \|T_j\|)$. Hence

$$SRx = \sum_{j=1}^{N} T_j P_j x = \sum_{j=1}^{N} T_j x = Tx,$$

SR is a factorization of T. Clearly $\|S\| \leqq N^{1/2} \leqq c(\ln(n+1))^{1/2}$. To estimate $\sigma_q(R)$, note that $\sum_{j=1}^k (\operatorname{rank} P_j) < 2^{k+2}$ for $k = 1, \cdots, N$. Hence

$$\alpha_{2^{k+2}}(R) \leqq \sup_{\|x\| \leqq 1} \Big(\sum_{j>k} \|T_j\|^2 \|P_j x\|^2 \Big)^{1/2} \leqq \Big(\sum_{j>k} \|T_j\|^2 \Big)^{1/2}.$$

Using the monotonicity of the approximation numbers, we get

$$\sigma_q(R) = \Big(\sum_{j=1}^{n} \alpha_j(R)^q \Big)^{1/q} \leqq c_1 \Big(\sum_{k=1}^{N} 2^k \alpha_{2^k}(R)^q \Big)^{1/q}$$

$$\leqq c_2 \Big(\sum_{k=1}^{N} 2^k \Big(\sum_{j>k} \|T_j\|^2 \Big)^{q/2} \Big)^{1/q}.$$

By a lemma of Pietsch [5], this is up to a constant

$$\leqq c_3 \Big(\sum_{j=1}^{N} 2^j \|T_j\|^q \Big)^{1/q}$$

$$\leqq c_4 \sigma_q(T) \leqq d_q \pi_{q,2}(T),$$

using (7), where the constants (may) depend only on q. Thus

$$\sigma_q(R) \|S\| \leqq c(\ln(n+1))^{1/2} \pi_{q,2}(T).$$

(b) To find operators T which do not factor well through $S_q(l_2)$, consider diagonal maps $D_\sigma : l_2^n \to l_1^n$, $(x_i)_{i=1}^n \mapsto (\sigma_i x_i)_{i=1}^n$. Assume $\sigma_1 \geqq \sigma_2 \geqq \cdots \geqq \sigma_n \geqq 0$. To calculate the $\pi_{q,2}$-norm of D_σ, we use

$$\alpha_j(D_\sigma) \leqq \Big(\sum_{k=j}^{n} \sigma_j^2 \Big)^{1/2} \qquad \text{for } j = 1, \cdots, n;$$

here in fact equality holds. Hence by Corollary 2

$$\pi_{q,2}(D_\sigma) \leqq c_q \sigma_q(D_\sigma) = c_q \Big(\sum_{j=1}^{n} \Big(\sum_{k=j}^{n} \sigma_k^2 \Big)^{q/2} \Big)^{1/q}.$$

The last norm, however, is equivalent to the norm $\|\cdot\|_{s,q}$ in the Lorentz sequence space $l_{s,q}$, $1/s = 1/q + 1/2$, that is

$$\pi_{q,2}(D_\sigma) \leqq d_q \|\sigma\|_{s,q} =: d_q \Big(\sum_{j=1}^{n} \sigma_j^q j^{q/2} \Big)^{1/q} \tag{8}$$

To see the equivalence observe that by the lemma of [5] and the monotonicity of the σ_j's one has the equivalences (for $n = 2^N$)

$$\left(\sum_{j=1}^{n}\left(\sum_{k=j}^{n}\sigma_k^2\right)^{q/2}\right)^{1/q} \sim \left(\sum_{i=1}^{N}2^i\left(\sum_{k=2^i}^{2N}\sigma_k^2\right)^{q/2}\right)^{1/q}$$

$$\sim \left(\sum_{i=1}^{N}2^i\left(\sum_{l=i}^{N}|2^l\sigma_{2^l}|^2\right)^{q/2}\right)^{1/q}$$

$$\sim \left(\sum_{i=1}^{N}2^i\,(2^i\sigma_{2^i})^{q/2}\right)^{1/q}$$

$$\sim \left(\sum_{k=1}^{n}k^{q/2}\sigma_k^{q/2}\right)^{1/q}.$$

Given an arbitrary factorization $D_\sigma = S_nR_n$, $R_n : l_2^n \to l_2$, $S_n : l_2 \to l_1^n$ of D_σ, consider the composition of D_σ with the inclusion map $I_n : l_1^n \to l_2^n$ which has 2-summing norm bounded independent of n. Hence, again with $1/s = 1/q + 1/2$

$$\begin{aligned}\|\sigma\|_s &= \sigma_s(I_nD_\sigma : l_2^n \to l_2^n) \leqq \sigma_q(R_n : l_2^n \to l_2^n)\sigma_2(I_nS_n : l_2^n \to l_2^n)\\ &= \sigma_q(R_n)\pi_2(I_nS_n) \leqq K_G\sigma_q(R_n)\|S_n\|.\end{aligned} \tag{9}$$

It is easy to see that there are sequences σ such that

$$\|\sigma\|_s \geqq c_1(\ln(n+1))^{1/2}\|\sigma\|_{s,q}.$$

Therefore (8) and (9) imply that there are diagonal maps $T_n = D_\sigma : l_2^n \to l_1^n$ with

$$\begin{aligned}\sigma_q(R_n)\|S_n\| &\geqq K_G^{-1}\|\sigma\|_s \geqq c_2(\ln(n+1))^{1/2}\|\sigma\|_{s,q}\\ &\geqq c_3(\ln(n+1))^{1/2}\pi_{q,2}(D_\sigma)\end{aligned}$$

for any factorization $T_n = S_nR_n$ of the above form.

Added in proof. Denoting the Rademacher type p and cotype q constants by $\tilde{K}^{(p,n)}(X)$ and $\tilde{K}_{(q,n)}(X)$, one has for n-dimensional spaces $X : \tilde{K}^{(p)}(X) \leqq c_p\tilde{K}^{(p,n)}(X)$ since the Gaussian and Rademacher type p constants are equivalent. Moreover, L. Tzafriri and the author have shown $\tilde{K}_{(q)}(X) \leqq c_q\tilde{K}_{(q,n)}(X)\sqrt{\log(\tilde{K}_{(q,n)}(X)+1)}$.

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